Final Report

OPTIMIZATION WITH PROBABILISTIC CONSTRAINTS

AFOSR Grant # FA9550-08-1-0117

Shabbir Ahmed
School of Industrial & Systems Engineering
Georgia Institute of Technology, Atlanta, GA 30332

Summary:

This report summarizes the outcome of the AFSOR grant FA9550-08-01-0117 during the project term 03/01/2008-12/31/2011.

Many important planning and design applications in uncertain environments involve service level or reliability requirements. These include emergency planning, telecommunication network design, cancer therapy planning, and financial optimization. Such requirements give rise to probabilistic or chance constraints. The stochasticity and nonconvexity associated with such constraints make the underlying optimization problem extremely challenging. Current approaches for probabilistically constrained optimization problems are either not able to handle realistic problems or provide much too conservative solutions. This project developed novel methods for this hard class of problems by combining ideas from integer programming and statistical analysis.

Results:

The three key outcomes of the project are the following.

1. *Sampling based approximations of probabilistic constraints:* We studied integer programming approximations of probabilistic constraints obtained by replacing the uncertain problem parameter by a set of iid samples. We established asymptotic convergence of these approximations and also schemes for bounding approximation quality from finite samples. The developed approach is very general and is applicable to a wide variety of chance constraint problems. A tutorial on this approached was given at INFORMS 2008.

2. *Probabilistic set covering problems with correlations:* Set covering problems are a very important class of problems arising in various applications. Many important applications, e.g. emergency response center location and sensor network design, give rise to set cover problems with uncertain coefficients. Exploiting the fact these coefficients are Bernoulli random variables, we very effective develop deterministic reformulations of these problems.

3. *Cutting planes for probabilistic constraints with coefficient uncertainties:* Solving integer programming approximations of probabilistic constraints is very difficult. There has been earlier work on developing methods for these problems when the uncertainty appears in the right-hand-side of the constraints. We extended these approaches to problems when constraint coefficients are uncertain. This is a much more difficult problem.

Personnel Supported:

The grant supported 1 summer month for the PI per year and 12 months for 1 graduate student, during the project term.

Publications:

S. Ahmed and D.J. Papageorgiou. "Probabilistic set covering with correlations," submitted to Operations Research, 2011.

S. Ahmed and A. Atamturk. "Maximizing a class of submodular utility functions," to appear in *Mathematical Programming*, 2011.

S. Ahmed and A. Shapiro. "Solving chance-constrained stochastic programs via sampling and integer programming," in *Tutorials in Operations Research*, Z.-L. Chen and S. Raghavan (eds.), INFORMS, 2008.

B. Pagnoncelli, S. Ahmed, and A. Shapiro. "The sample average approximation method for chance constrained programming: theory and applications," *Journal of Optimization theory and Applications*, vol.142, pp.399-416, 2009.

F. Qiu, S. Ahmed and S.S. Dey. "Cutting planes for probabilistic constraints with uncertain coefficients," working paper, 2011.

S. Shen, J.C. Smith, and S. Ahmed. "Expectation and Chance-Constrained Models and Algorithms for Insuring Critical Paths," *Management Science*, vol.56, pp.1794-1814, 2010.

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14. ABSTRACT

Many important planning and design applications in uncertain environments involve service level or reliability requirements. These include emergency planning, telecommunication network design, cancer therapy planning, and financial optimization. Such requirements give rise to probabilistic or chance constraints. The stochasticity and nonconvexity associated with such constraints make the underlying optimization problem extremely challenging. Current approaches for probabilistically constrained optimization problems are either not able to handle realistic problems or provide much too conservative solutions. In this work: (i) We integrated sampling theory with mixed-integer programming schemes to effectively and efficiently solve large classes of such problems. (ii) We developed new formulations for a wide class of probabilistic set covering problems by exploiting sumodularity properties. (iii) We developed new algorithmic techniques for probabilistic constraints with coefficient uncertainties.

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